Algebra 2 Course, Unit 2 – Worksheet 13 – Overview of Systems of Linear Equations

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1. Is (-4, -2) a solution to the system of equations below? Show your work.

$$\begin{cases} -3x + 2y = 8\\ x + 2y = -8 \end{cases}$$

2. Is (-2,1) a solution to the system of equations below? Show your work.

$$\begin{cases} x - 2y = 4\\ 3x + y = 5 \end{cases}$$

3. Is (20,52) a solution to the system of equations below? Show your work.

$$\begin{cases} y = 0.75x + 37 \\ y = 1.5x + 22 \end{cases}$$

4. Is (4,2) a solution to the system of equations below? Show your work.

$$\begin{cases} y = x - 2\\ 2x + y = 7 \end{cases}$$

5. Is (2,1) a solution to the system of equations below? Show your work.

$$\begin{cases} x + y = 3\\ 2y = 3x - 4 \end{cases}$$

6. Is (-13,4) a solution to the system of equations below? Show your work.

$$\begin{cases} 4y = -x + 3\\ x + 4y = 3 \end{cases}$$

$$\begin{cases} 4y - 2x = 6\\ 8y = 4x - 12 \end{cases}$$

8. Without solving, determine whether the system of equations below has one solution, many solutions, or no solution. Justify your answer.

$$\begin{cases} -3x + y = 4\\ x - \frac{1}{3}y = 1 \end{cases}$$

9. Without solving, determine whether the system of equations below has one solution, many solutions, or no solution. Justify your answer.

$$\begin{cases} 2x + 3y = 1\\ 4x + y = -3 \end{cases}$$

$$\begin{cases} y = 2x - 3\\ 6x - 3y = 9 \end{cases}$$

11. Without solving, determine whether the system of equations below has one solution, many solutions, or no solution. Justify your answer.

$$\begin{cases} 2x + 4y = 12\\ x + y = 2 \end{cases}$$

12. Without solving, determine whether the system of equations below has one solution, many solutions, or no solution. Justify your answer.

$$\begin{cases} 3x - y = 5\\ x - y = 7 \end{cases}$$

Answers – Algebra 2 Course, Unit 2 – Worksheet 13 – Overview of Systems of Linear Equations

To determine if a point (x_1, y_1) is a solution to a system of equations, substitute x_1 into both equations for x and y_1 into both equations for y. If the result is two identity equations, then the point is a solution to the system.

1. Is (-4, -2) a solution to the system of equations below? Show your work.

$$\begin{cases} -3x + 2y = 8\\ x + 2y = -8 \end{cases}$$

$$-3x + 2y = 8; \quad -3(-4) + 2(-2) = 8?; \quad 12 - 4 = 8? \quad 8 = 8$$

$$x + 2y = -8; \quad -4 + 2(-2) = -8?; \quad -4 + (-4) = -8?; \quad -8 = -8$$

Answer: Yes, the point satisfies both equations.

2. Is (-2,1) a solution to the system of equations below? Show your work.

$$\begin{cases} x - 2y = 4\\ 3x + y = 5 \end{cases}$$

$$x - 2y = 4; \ -2 - 2(1) = 4?; \ -2 - 2 = 4?; \ -4 \neq 4$$

$$3x + y = 5; \ 3(-2) + 1 = 5?; \ -6 + 1 = 5?; \ -5 \neq 5 \end{cases}$$

Answer: No, the point does not satisfy either of the equations.

3. Is (20,52) a solution to the system of equations below? Show your work.

$$\begin{cases} y = 0.75x + 37 \\ y = 1.5x + 22 \end{cases}$$

$$y = 0.75x + 37; \quad 52 = 0.75(20) + 37?; \quad 52 = 15 + 37?; \quad 52 = 52$$

$$y = 1.5x + 22; \quad 52 = 1.5(20) + 22?; \quad 52 = 30 + 22?; \quad 52 = 52$$

Answer: Yes, the point satisfies both equations.

4. Is (4,2) a solution to the system of equations below? Show your work.

$$\begin{cases} y = x - 2\\ 2x + y = 7 \end{cases}$$

$$y = x - 2; \quad 2 = 4 - 2?; \quad 2 = 2$$

$$2x + y = 7; \quad 2(4) + 2 = 7?; \quad 8 + 2 = 7?; \quad 10 \neq 7 \end{cases}$$

Answer: No, the point satisfies only one of the equations.

5. Is (2,1) a solution to the system of equations below? Show your work.

$$\begin{cases} x + y = 3\\ 2y = 3x - 4 \end{cases}$$
$$x + y = 3; \quad 2 + 1 = 3?; \quad 3 = 3$$
$$2y = 3x - 4; \quad 2(1) = 3(2) - 4?; \quad 2 = 6 - 4?; \quad 2 = 2 \end{cases}$$

Answer: Yes, the point satisfies both equations.

6. Is (-13,4) a solution to the system of equations below? Show your work.

$$\begin{cases} 4y = -x + 3\\ x + 4y = 3 \end{cases}$$

$$4y = -x + 3; \quad 4(4) = -(-13) + 3?; \quad 16 = 13 + 3?; \quad 16 = 16$$

$$x + 4y = 3; \quad -13 + 4(4) = 3?; \quad -13 + 16 = 3?; \quad 3 = 3 \end{cases}$$

Answer: Yes, the point satisfies both equations.

To determine the type of solution a system of equations has, solve for y in both equations so the equations are in the form y = mx + b.

If the slopes (m) are not the same, the lines intersect, so the system has one solution.

If the slopes (m) are the same and the y-intercepts (b) are not the same, the lines are parallel, so the system has no solutions.

If both the slopes (m) and the y-intercepts (b) are the same, the equations represent the same line, so the system has infinitely many solutions.

7. Without solving, determine whether the system of equations below has one solution, many solutions, or no solution. Justify your answer.

$$\begin{cases} 4y - 2x = 6\\ 8y = 4x - 12 \end{cases}$$

$$4y - 2x = 6; \quad 4y - 2x + 2x = 2x + 6$$

$$4y = 2x + 6; \quad \frac{4y}{4} = \frac{2x}{4} + \frac{6}{4}$$

$$y = \frac{1}{2}x + \frac{3}{2}; \quad m = \frac{1}{2}, b = \frac{3}{2}$$

$$8y = 4x - 12; \ \frac{8y}{8} = \frac{4x - 12}{8}; \ \frac{8y}{8} = \frac{4x}{8} - \frac{12}{8}$$
$$y = \frac{1}{2}x - \frac{3}{2}; \ m = \frac{1}{2}, b = -\frac{3}{2}$$

Answer: No solution, the lines are parallel.

$$\begin{cases} -3x + y = 4\\ x - \frac{1}{3}y = 1 \end{cases}$$

-3x + y = 4; -3x + y + 3x = 3x + 4
y = 3x + 4; m = 3, b = 4

$$x - \frac{1}{3}y = 1; \ x - \frac{1}{3}y - x = -x + 1$$

$$-\frac{1}{3}y = -x + 1; \ -\frac{3}{1}\left(-\frac{1}{3}y\right) = -\frac{3}{1}(-x + 1)$$

$$y = -3(-x) - 3(1)$$

$$y = 3x - 3; \ m = 3, b = -3$$

Answer: No solution, the lines are parallel.

 $\begin{cases} 2x + 3y = 1\\ 4x + y = -3 \end{cases}$ $2x + 3y = 1; \ 2x + 3y - 2x = -2x + 1$ $3y = -2x + 1; \ \frac{3y}{3} = \frac{-2x + 1}{3}$ $y = -\frac{2}{3}x + \frac{1}{3}; \ m = -\frac{2}{3}, b = \frac{1}{3}$

$$4x + y = -3; \ 4x + y - 4x = -4x - 3$$
$$y = -4x - 3; \ m = -4, b = -3$$

Answer: One solution, the lines intersect.

10. Without solving, determine whether the system of equations below has one solution, many solutions, or no solution. Justify your answer.

$$\begin{cases} y = 2x - 3\\ 6x - 3y = 9 \end{cases}$$

y = 2x - 3; m = 2, b = -3

$$6x - 3y = 9; \ 6x - 3y - 6x = -6x + 9$$
$$-3y = -6x + 9; \ \frac{-3y}{-3} = \frac{-6x + 9}{-3}; \ y = \frac{-6x}{-3} + \frac{9}{-3}$$
$$y = 2x - 3; \ m = 2, b = -3$$

Answer: Infinitely many solutions, the equations represent the same line.

$$\begin{cases} 2x + 4y = 12\\ x + y = 2 \end{cases}$$

$$2x + 4y = 12; \quad 2x + 4y - 2x = -2x + 12$$

$$4y = -2x + 12; \quad \frac{4y}{4} = \frac{-2x + 12}{4}$$

$$y = -\frac{2x}{4} + \frac{12}{4}; \quad y = -\frac{1}{2}x + 3; \quad m = -\frac{1}{2}, b = 3$$

$$x + y = 2; \quad x + y - x = -x + 2$$

Answer: One solution, the lines intersect.

12. Without solving, determine whether the system of equations below has one solution, many solutions, or no solution. Justify your answer.

y = -x + 2; m = -1, b = 2

$$\begin{cases} 3x - y = 5\\ x - y = 7 \end{cases}$$

$$3x - y = 5; \ 3x - y - 3x = -3x + 5$$

$$-y = -3x + 5; \ \frac{-y}{-1} = \frac{-3x + 5}{-1}; \ y = \frac{-3x}{-1} + \frac{5}{-1}$$

$$y = 3x - 5; \ m = 3, b = -5$$

$$x - y = 7; \ x - y - x = -x + 7$$

$$-y = -x + 7; \ \frac{-y}{-1} = \frac{-x}{-1} + \frac{7}{-1}$$

$$y = x - 7; \ m = 1, b = -7$$

Answer: One solution, the lines intersect.